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Tugas Asistensi Matematika 2 (Kode : B)

1. Diberikan fungsi $y = 1 - \ln(ax)$ dengan a = digit terakhir NRP

(a). Apakah fungsi tersebut punya Invers?

→ PUNYA.

- Berdasarkan Grafik

Semua nilai x memiliki tepat satu pasangan pada y. Begitupun Sebaliknya.

→ Tak ada satupun nilai y yang didefinisikan oleh > 1 nilai x.

(FUNGSI BIJEKTIF).

- Berdasarkan Numerik

$$f^{-1}(f(x)) = x$$
$$f(f^{-1}(x)) = x$$

* Invers dari $y = 1 - \ln(2x)$

$$y - 1 = -\ln 2x$$
$$\ln 2x = 1 - y$$
$$\ln 2x = (1-y) . \ln e$$
$$\ln 2x = \ln e^{1-y}$$
$$2x = e^{1-y}$$
$$x = \frac{e^{1-y}}{2}$$

Lakukan Pembuktian :

$$f^{-1}(f(x)) = \frac{e^{1-f(x)}}{2}$$
$$= \frac{e^{1-(1-\ln 2x)}}{2}$$
$$= \frac{e^{\ln 2x}}{2}$$
$$= \frac{e^{{}^{e}\log 2x}}{2}$$
$$= \frac{2x}{2}$$
$$= x$$

$$f(f^{-1}(x)) = 1 - \ln 2 f^{-1}(x)$$
$$= 1 - \ln \cancel{2} . \frac{e^{1-x}}{\cancel{2}}$$
$$= 1 - \ln e^{1-x}$$
$$= 1 - (1-x)$$
$$= x$$

(TERBUKTI)

b. Jika punya Invers, dapatkan Invers beserta domain dan rangenya.

Invers → $\boxed{f^{-1}(x) = \frac{e^{1-x}}{2}}$

Untuk menentukan domain dan range dari invers suatu fungsi, kita bisa tentukan melalui domain dan range fungsi awal.

Domain $f(x) \longrightarrow (0, +\infty)$

Range $f(x) \longrightarrow (-\infty, +\infty)$

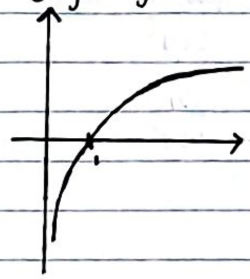
Maka, domain dan range $f^{-1}(x)$ adalah

Domain $f^{-1}(x) \longrightarrow (-\infty, +\infty)$

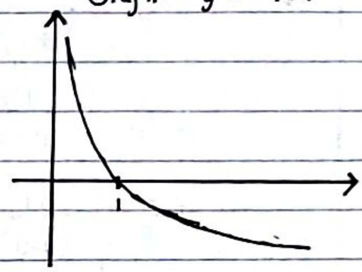
Range $f^{-1}(x) \longrightarrow (0, +\infty)$

c. Sketsa fungsi y dan invers dalam satu bidang koordinat.

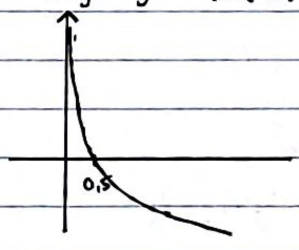
- Grafik $y = \ln x$



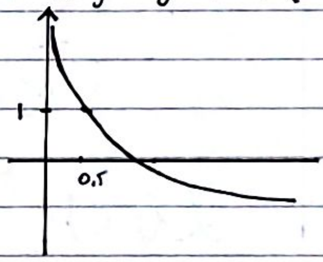
- Grafik $y = -\ln x$



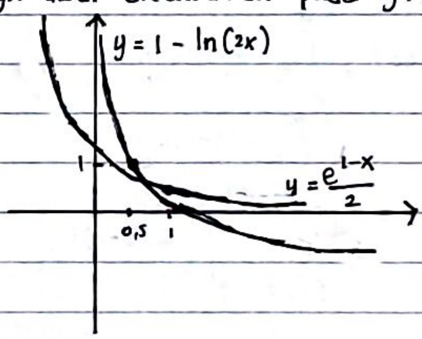
- Grafik $y = -\ln(2x)$



- Grafik $y = 1 - \ln(2x)$



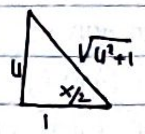
* Grafik invers dari suatu fungsi adalah grafik fungsi awal dicerminkan pada garis $y = x$



2. $\phi = \int \frac{1}{5+5\sin x}\,dx$

$= \frac{1}{5}\int \frac{1}{1+\sin x}\,dx$

misal : $u = \tan\left(\frac{x}{2}\right)$



$\tan^{-1} u = \frac{x}{2}$

$2\tan^{-1} u = x$

$\frac{2}{1+u^2}\,du = dx$

$\sin\frac{x}{2} = \frac{u}{\sqrt{u^2+1}}$

$\cos\frac{x}{2} = \frac{1}{\sqrt{u^2+1}}$

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= 2 \cdot \frac{u}{\sqrt{u^2+1}} \cdot \frac{1}{\sqrt{u^2+1}}$

$= \frac{2u}{u^2+1}$

$\rightarrow \phi = \frac{1}{5}\int \frac{1}{1+\frac{2u}{u^2+1}} \cdot \frac{2\,du}{1+u^2}$

$= \frac{2}{5}\int \frac{\cancel{u^2+1}}{u^2+2u+1}\,\frac{du}{\cancel{1+u^2}}$

$= \frac{2}{5}\int \frac{du}{(u+1)^2}$

$= \frac{2}{5}\left[-\frac{1}{u+1}\right] + C$

$= \frac{-2}{5\tan\frac{x}{2}+5} + C$ //

3. $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

a. Dapatkan $\mathcal{L}\{e^{at}\}$ dimana a digit terakhir NRP.

$\mathcal{L}\{e^{2t}\} = \int_0^\infty e^{-st} \cdot e^{2t}\,dt$

$= \int_0^\infty e^{-(s-2)t}\,dt$

$= \lim_{p\to\infty} \int_0^p e^{-(s-2)t}\,dt$

$= \frac{-1}{(s-2)} \cdot \lim_{p\to\infty} e^{-(s-2)t}\Big]_0^p$

$= -\frac{1}{s-2} \lim_{p\to\infty}\left[e^{-2p} - e^0\right]$

$= -\frac{1}{s-2}\left[0-1\right]$

$= \frac{1}{s-2}, \ s>2$ //

b. $\mathcal{L}\{\sin\omega t\}$, di mana ω adalah konstanta

$\rightarrow \mathcal{L}\{\sin\omega t\} = \int_0^\infty e^{-st} \cdot \sin\omega t\,dt$

※ Hubungan Euler

$e^{ix} = \cos x + i\sin x \ ; \ e^{-ix} = \cos x - i\sin x$

$\rightarrow e^{ix} - e^{-ix} = (\cos x + i\sin x) - (\cos x - i\sin x)$

$e^{ix} - e^{-ix} = 2.i\sin x$

$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$

$\Rightarrow \mathcal{L}\{\sin\omega t\} = \int_0^\infty \frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right)e^{-st}\,dt$

$= \frac{1}{2i}\left[\int_0^\infty \left(e^{-(s-i\omega)t}\right)dt - \int_0^\infty e^{-(s+i\omega)t}\,dt\right]$

$= \frac{1}{2i}\left(\frac{1}{-(s-i\omega)} e^{-(s-i\omega)t}\Big|_0^\infty - \frac{1}{-(s+i\omega)} e^{-(s+i\omega)t}\Big|_0^\infty\right)$

$= \frac{1}{2i}\left(\left[0 + \frac{1}{s-i\omega}\right] - \left[0 + \frac{1}{s+i\omega}\right]\right)$

$= \frac{1}{2i}\left(\frac{1}{s-i\omega} - \frac{1}{s+i\omega}\right)$

$= \frac{1}{2i}\left(\frac{s+i\omega-(s-i\omega)}{s^2+\omega^2}\right)$

$= \frac{1}{\cancel{2i}}\left(\frac{\cancel{2i}\,\omega}{s^2+\omega^2}\right)$

$\mathcal{L}\{\sin\omega t\} = \frac{\omega}{s^2+\omega^2}$ //

4). $\lim_{x\to 0^+} \frac{\cot x}{\ln x}$

▶ Substitusi langsung untuk cek apakah menghasilkan bentuk tak tentu.

$\frac{\cot 0}{\ln 0} = \frac{\infty}{-\infty}$ (tak tentu).

Gunakan dalil L'Hopital

$$\lim_{x\to 0^+} \frac{\cot x}{\ln x} = \lim_{x\to 0^+} \frac{-\operatorname{cosec}^2 x}{1/x}$$

$$= \lim_{x\to 0^+} -x \operatorname{cosec}^2 x$$

$$= \lim_{x\to 0^+} -\frac{x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -1 \cdot \lim_{x\to 0^+} \frac{1}{\sin x}$$

$$= -1 \cdot \infty$$

$$= -\infty$$

5. Tunjukkan bahwa luas Elips adalah $A = \pi ab$ jika persamaan elips adalah

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$$

a, b adalah puncak elips

$$\rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

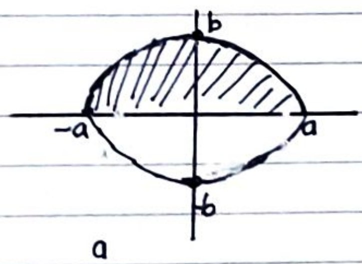
$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \quad \times b^2$$

$$y^2 = b^2 - \frac{b^2}{a^2}x^2$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

$$y = \pm b\sqrt{\frac{a^2 - x^2}{a^2}}$$

$$= \pm \frac{b}{a}\sqrt{a^2 - x^2}$$



$a > b$

$$L_{arsir} = \int_{-a}^{a} \frac{b}{a}\sqrt{a^2 - x^2}\, dx$$

misal : $x = a \sin\theta$

$dx = a \cos\theta \, d\theta$

Batas bawah $\longrightarrow -a = a \sin\theta$

$\sin\theta = -1$

$\theta = -\frac{\pi}{2}$

Batas Atas $\longrightarrow a = a \sin\theta$

$\sin\theta = 1$

$\theta = \pi/2$

$$\rightarrow \frac{b}{\cancel{a}} \int_{-\pi/2}^{\pi/2} \sqrt{a^2 - a^2\sin^2\theta} \cdot \cancel{a} \cos\theta \, d\theta$$

$$= b \int_{-\pi/2}^{\pi/2} a\sqrt{1 - \sin^2\theta} \cdot \cos\theta \, d\theta$$

$$= ab \int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta$$

$$= ab\left[\int_{-\pi/2}^{\pi/2} \frac{\cos 2\theta}{2} d\theta + \int_{-\pi/2}^{\pi/2} \frac{1}{2} d\theta\right]$$

$$= \frac{ab}{2}\left(\left[-\frac{1}{2}\sin 2\theta\right]_{-\pi/2}^{\pi/2} + \left[\theta\right]_{-\pi/2}^{\pi/2}\right)$$

$$= \frac{ab}{2}\left(\left[\left(\left(-\tfrac{1}{2}\cdot 0\right) - \left(-\tfrac{1}{2}\cdot 0\right)\right)\right] + \left[\left(\tfrac{\pi}{2} - \left(-\pi/2\right)\right)\right]\right)$$

$$= \frac{ab}{2}(0 + \pi)$$

$$= \frac{\pi ab}{2}$$

$$L_{elips} = A = 2 \cdot L_{arsir}$$

$$= \cancel{2} \cdot \frac{\pi ab}{\cancel{2}}$$

$$= \pi ab$$

Jadi, luas elips $A = \pi ab$